Profitability For Startup Business

Graham Ellinson

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1 The Business Model

A business entrepreneur plans to start a new venture delivering fresh organic food to restaurants and private households. The startup costs are estimated to be $\$$40,000, which includes the purchase of a secondhand delivery van, securing storage space, the procurement of suitable packaging and advertising. The money will be borrowed as an unsecured loan at a nominal interest rate of 6% per annum, to be repaid monthly in arrear over a period of five years.

The business model is to harvest in-season produce from local farmers and sell them in units classed as small and large. A small unit will sell at £15 and a large unit at £25, with a delivery cost of £15 added to each unit. An order of 10 large units and 20 small units have been secured, with an expected growth in orders of 5% and 16% per month for the small and large units respectively. It is estimated that market saturation will be 80 large units and 100 small units per month. All income from sales can be assumed to be received continuously during each month.

The running costs of the business will be £1,000 per month, excluding salaries. Staff will be paid the national minimum wage per month, so that the monthly cost to the business is £1,480 per person. The business will employ 2 people, with an annual salary increase of 3% implemented at the start of each year of employment.

2 Fixed Monthly Premiums

We begin by calculating the fixed monthly premiums.

The interest rate is 6% per annum, however as this problem is calculating monthly premiums then interest per payment period is $\frac{0.06}{12} = 0.005$.

So monthly premiums are:

$$40,000 = xv + xv^2 + xv^3 + ... + xv^{60}, \tag{1}$$

where V is a function of the rate of interest i,

$$v = (1 + i)^{-1}.$$

However, Equation (1) can be expressed in terms of an annuity $a_{\overline{n}|}$ where:

$$a_{\overline{n}|} = v + v^2 + v^3 + ... + v^n,$$

this is also written as:

$$v = \frac{1 - v^n}{i}.$$

Hence 1 can be written as:

$$40,000 = xa_{\overline{n}|},$$
$$40,000 = x\frac{1 - v^n}{i}.$$

Inputting the values for i, v gives:

$$40,000 = x\left[\frac{1 - [(1 + 0.005)^{-1}]^{60}}{0.005}\right]$$

$$40,000 = x51.72556075,$$

so

$$x = \frac{40,000}{51.72556075}$$
$$x = 773.3120612.$$

Hence the fixed monthly premiums to repay the loan within the predetermined term of five years are $\pounds$773.31.

3 Yield and Profitability for a Five Year Project

To enable us to properly model the net present value of the proposed business, we will calculate the yield and determine the profitability for a project duration of 5 years.

In order to write an expression for the net present value of the proposed business it is necessary to first ascertain and estimate the net cash flows of the business. To do this it is important to evaluate all the outgoings and estimated expenses for the business together with the income and estimated revenue.

Outgoings

The discrete outgoings for the proposed business is as follows:

1. $\pounds$40,000 for the startup costs (delivery van, storage space, packaging and advertising).

2. $\pounds$773.31 per month for the duration of the five years to repay the loan.

3. $\pounds$1,000 per month running cost.

4. £2,960 per month for the fist year for salaries $1,480 for two members of staff.
5. £3,048.80 per month in salaries for the second year (3% increase on from year 1).
6. £3,140.26 per month in salaries for the third year (3% increase on from year 2).
7. £3,234.47 per month in salaries for the fourth year (3% increase on from year 3).
8. £3,331.51 per month in salaries for the fifth year (3% increase on from year 4).

These values (except for the initial £40,000) can be totalled and tabulated as the following outgoings per month:

Year 1:	£4,733.31
Year 2:	£4,822.11
Year 3:	£4,913.57
Year 4:	£5,007.78
Year 5:	£5,101.82

Incomings

There is only one discrete income into this business and that is the £40,000 obtained from the loan since this was spent immediately as explained above in the outgoings this £40,000 is not included going forward within the project evaluation.

The only source of income considered here in this business model, is the income from the large and small units that are sold. This income is treated as continuously paid into the business between time interval (0, T). It can be expressed as a function at a rate of payment £$\rho(t)$ per unit time. Small units sell at £15 + £15 delivery and grows at a rate of 5% per month. Thus $\rho(t)_S$ is:

$$\rho(t)_S = n_0 \times 30 \times (1.05)^t,$$

with n_0 = 20 (the number of units initially ordered) and t being the point in time which is being evaluated. However, as there is a market saturation of 100 small units then $20 \times (1.05)^t \leq 100$ is rearranged to find t = 32.987 meaning from the 33rd month it will have reached market saturation. Thus $\rho(t)_S$ is now expressed in two parts

$$\begin{aligned}\rho(t)_S &= 600 \times (1.05)^t \; for \; t \; \leq \; 32 \; \& \\ &= 3000 \; for \; t > 32.\end{aligned}$$

Large units sell at £25 + £15 delivery and grows at a rate of 16% per month. Thus $\rho(t)_L$ is:

$$\rho(t)_L = n_0 \times 40 \times (1.16)^t,$$

with n_0 = 10 (the number of units initially ordered) and t being the point in time which is being evaluated. However, as there is a market saturation of 80 large units then $10 \times (1.16)^t \leq 80$ is rearranged to find t = 14.0105 meaning from the 15th month it will have reached market saturation. Thus $\rho(t)_L$ is now expressed in two parts

$$\begin{aligned}\rho(t)_L &= 400 \times (1.16)^t \; for\, t \; \leq \; 14 \; \& \\ &= 3200 \; for\, t > 14.\end{aligned}$$

Net Present Value of the Proposed Business

Assuming that the business may borrow or lend at a fixed rate of interest i per unit time. Then by the time the project ends the balance the account will be (Remer and Nieto, 1995)

$$C = \sum_t c_t(1+i)^{T-t} + \int_0^T \rho(t)(1+i)^{T-t}dt.$$

The present value for a period of time T is $PV = \frac{C}{(1+i)^T}$. Consequently, the present rate of interest i of the net present value at rate of interest i is

$$NPV(i) = \sum_t c_t(1+i)^{-t} + \int_0^T \rho(t)(1+i)^{-t}dt. \tag{2}$$

Inputting the net incomings and outgoings from above with unit of time expressed in months gives the following net present value equation for the proposed business:

$$\begin{aligned}NPV(i) = &-\sum_{t=1}^{12} 4733.31(1+i)^{-t} - \sum_{t=13}^{24} 4822.11(1+i)^{-t} \\ &- \sum_{t=25}^{36} 4913.57(1+i)^{-t} - \sum_{t=37}^{48} 5007.78(1+i)^{-t} \\ &- \sum_{t=49}^{60} 5104.82(1+i)^{-t} + \int_0^{32} 600(1.05)^t(1+i)^{-t}dt \\ &+ \int_{32}^{60} 3000^t(1+i)^{-t}dt + \int_0^{14} 400(1.16)^t(1+i)^{-t}dt \\ &+ \int_{14}^{60} 3200^t(1+i)^{-t}dt.\end{aligned} \tag{3}$$

Equation (3) is an equation of value for the business at present time if $NPV(i) = 0$, the yield i_0 is the solution of this equation. Using the technical computing system Mathematica® to find the root for Equation (3) by applying the bisection method (see Appendix A). Gives the root of $NPV(i_0)$ as i_0 = 0.00108538.

However, given as the unit of time in Equation (3) is months the yield for a project that has units of time in years is $0.00108538 \times 12 = 0.01302456$. This gives a yield of i_0 = 1.3%.

Interpreting the Yield and Determining the Profitability

Suppose that the business earns interest at a fixed rate i_1 then from equation (3), the business will only be profitable if and only if

$$NPV(i_1) > 0$$

Furthermore the project will only be profitable if and only if $i_1 < i_0$ i.e. the yield exceeds the rate of interest that the business may lend or borrow money. (Remer and Nieto, 1995)

The yield here is 1.3% meaning an interest rate that exceeds this values of 1.3% will mean the business will make a loss. For the business to make a profit within this project time of five years then the fixed interest rate for borrowing and lending must be below or equal to 1.3%.

4 Discounted Payback Period

If i_1 is the rate of interest applicable to the businesses borrowings then the balance of the business at time τ, $A(\tau)$, is (Remer and Nieto, 1995)

$$A(\tau) = \sum_{t\tau} c_t(1+i_1)^{\tau-t} + \int_0^{\tau} \rho(t)(1+i_1)^{\tau-t}dt. \tag{4}$$

The discounted payback period, τ_1, is the smallest value of τ such that $A(\tau) \geq 0$. Here in this business model the fixed rate of interest for borrowing is assumed to remain fixed at 6% and for a monthly time interval this is $i_1 = 0.005$ as calculated in section 1.1. However, as τ_1 is unknown then equation (4) therefore varies depending on the year in which it lies. For example if the discounted payback period lies within the first year then equation (4) becomes

$$\begin{aligned} A(\tau_1) &= -\sum_{t=1}^{\tau_1} 4733.31(1+i_1)^{\tau_1-t} + \int_0^{\tau_1} 600(1.05)^t(1+i_1)^{\tau_1-t}dt \\ &= \int_0^{\tau_1} 400(1.16)^t(1+i_1)^{\tau_1-t}dt, \end{aligned} \tag{5}$$

with i_1 = 0.005. The remaining terms that are in the original NPV equation however only apply after the first year and are therefore excluded for finding the root for Equation (5). Using Mathematica® (See Appendix A) to find the value of τ_1 it found a solution in the fourth year. Subsequently, Equation (4) is expressed as

$$\begin{aligned} A(\tau_1) = &-\sum_{t=1}^{12} 4733.31(1+i_1)^{-t} - \sum_{t=13}^{24} 4822.11(1+i)^{-t} \\ &-\sum_{t=25}^{36} 4913.57(1+i)^{-t} - \sum_{t=37}^{\tau_1} 5007.78(1+i)^{\tau_1 - t} \\ &+\int_0^{32} 600(1.05)^t (1+i_1)^{-t} dt + \int_{32}^{\tau_1} 3000^t (1+i)^{\tau_1 - t} dt \\ &+\int_0^{14} 400(1.16)^t (1+i_1)^{-t} dt + \int_{14}^{\tau_1} 3200^t (1+i)^{\tau_1 - t} dt \end{aligned} \tag{6}$$

The solution for this is τ_1 = 43.1811 which means that the discounted payback period occurs in month 44 which is the 8th month of the 4th year to the nearest month.

5 Extending the Project to Six Years

To extend the project to six years the incomings and outgoings for the extra year must first be calculated. Assuming that the loan period remains at 5 years, these outgoings of £773.31 are no longer being deducted for the six year. Additionally, it is important to note that the market saturation has already been reached within the five year period so there are no more orders than 80 large units and 100 large units within this extra year. The salary for the two employees are now £3431.46 per month due to the 3% increase from year five. The running costs remain the same at £1,000 making the total outgoings per month for the sixth year £4431.46 and the incomings only require a change in the upper limit of the integral by a further 12 months. Inputting these extra values means that equations 3 now becomes

$$\begin{aligned} NPV(i) = &-\sum_{t=1}^{12} 4733.31(1+i)^{-t} - \sum_{t=13}^{24} 4822.11(1+i)^{-t} \\ &-\sum_{t=25}^{36} 4913.57(1+i)^{-t} - \sum_{t=37}^{48} 5007.78(1+i)^{-t} \\ &-\sum_{t=49}^{60} 5104.82(1+i)^{-t} - \sum_{t=61}^{72} 4431.46(1+i)^{-t} \\ &+\int_0^{32} 600(1.05)^t (1+i)^{-t} dt + \int_{32}^{72} 3000^t (1+i)^{-t} dt \\ &+\int_0^{14} 400(1.16)^t (1+i)^{-t} dt + \int_{14}^{72} 3200^t (1+i)^{-t} dt. \end{aligned} \tag{7}$$

Using the bisection method with Mathematica® (See Appendix A) the root for Equation (7) is now i_0 = 0.0117592. Accounting for time units of years give i_0 = 0.0117592 $\times$ 12 = 0.1411104, which gives a yield of 14.11%.

This means that extending the project by one year gives a much larger room for profit and the margin at which the interest will be too high for the business to be profitable is now much larger.

6 The Sustainability and Assumptions of the Business Model

The difference between a project time of five years versus a time-frame of six years illustrates just how much more of an impact it has on the sustainability of the business model. With the shorter project of five years there is fairly small room to actually make a profit due to a relatively small yield. However, the six year project has a far larger yield and therefore the rate of return is far larger.

There are a number of assumptions made here in this business model. The assumption that the initial startup costs for purchase of delivery van, packaging and other costs will not need replenishing or replacing within the project time-frame. Similarly, although it states that the running costs of the business are $1,000 it does not appear to factor in the costs of purchasing the in-season produce from local farmers. This is a point that can have significant impact on the sustainability of the model as often crop yields vary and the cost of purchasing produce can depend heavily on the size of each harvest.

Further assumptions made in this model are, that there is no taxation, inflation and that the volume and growth of sales will proceed as expected. Additionally, within the net present value context it is assumed that there is a fixed rate of interest for both borrowing and lending.

References

D. S. Remer and A. P. Nieto. A compendium and comparison of 25 project evaluation techniques. part 1: Net present value and rate of return methods. *International journal of production economics*, 42(1):79–96, 1995.

Appendix A

Net Present Value Bisection Method for Five Year Project

In[∘]:=
$$NPV = -\sum_{t=1}^{12}\left(4733.31*(1+i)^{-t}\right)-\sum_{t=13}^{24}\left(4822.11*(1+i)^{-t}\right)-\sum_{t=25}^{36}\left(4913.57*(1+i)^{-t}\right)-\sum_{t=37}^{48}\left(5007.78*(1+i)^{-t}\right)-\sum_{t=49}^{60}\left(5104.82*(1+i)^{-t}\right)+\int_{0}^{32}600*1.05^{t}*(1+i)^{-t}\,dt+\int_{32}^{60}3000*(1+i)^{-t}\,dt+\int_{0}^{14}400*1.16^{t}*(1+i)^{-t}\,dt+\int_{14}^{60}3200*(1+i)^{-t}\,dt;$$

```
In[∘]:= i = 0.003;

In[∘]:= Solve[NPV]
Out[∘]= Solve[-2211.]

In[∘]:= i = 0.001;

In[∘]:= Solve[NPV]

In[∘]:= FindRoot[NPV == 0, {i, 0.001, 0.003}]
Out[∘]= {0.001 → 0.00108538}
```

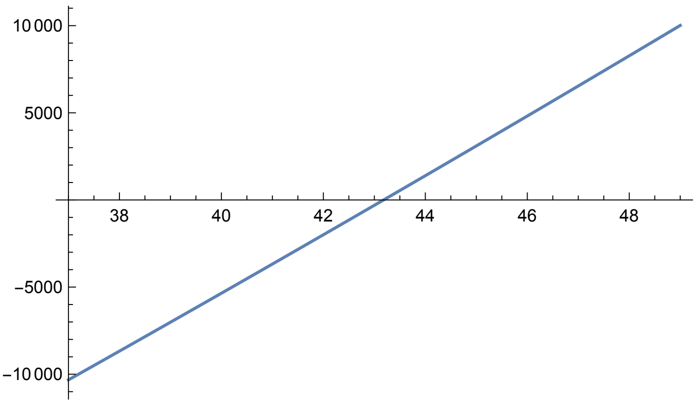
Discounted Payback Period Solution

In[∘]:=
$$A4 = -\sum_{t=1}^{12}\left(4733.31*(1+i)^{-t}\right)-\sum_{t=13}^{24}\left(4822.11*(1+i)^{-t}\right)-\sum_{t=25}^{36}\left(4913.57*(1+i)^{-t}\right)-\sum_{t=37}^{\tau}\left(5007.78*(1+i)^{\tau-t}\right)+\int_{0}^{32}600*1.05^{t}*(1+i)^{-t}\,dt+\int_{32}^{\tau}3000*(1+i)^{\tau-t}\,dt+\int_{0}^{14}400*1.16^{t}*(1+i)^{-t}\,dt+\int_{14}^{\tau}3200*(1+i)^{\tau-t}\,dt;$$

```
In[∘]:= i = 0.005;

In[∘]:= Plot[A4, {τ, 37, 49}]
```

Out[∘]=



```
In[•]:= FindRoot[A4, {τ, 43}]
Out[•]= {τ → 43.1811}
```

Net Present Value Bisection Method for Six Year Project

In[1]:=

$$NPV = -\sum_{t=1}^{12}\left(4733.31*(1+i)^{-t}\right) - \sum_{t=13}^{24}\left(4822.11*(1+i)^{-t}\right) - \sum_{t=25}^{36}\left(4913.57*(1+i)^{-t}\right) - \sum_{t=37}^{48}\left(5007.78*(1+i)^{-t}\right) - \sum_{t=49}^{60}\left(5104.82*(1+i)^{-t}\right) - \sum_{t=61}^{72}\left(4431.46*(1+i)^{-t}\right) + \int_0^{32} 600*1.05^t*(1+i)^{-t}\,dt + \int_{32}^{72} 3000*(1+i)^{-t}\,dt + \int_0^{14} 400*1.16^t*(1+i)^{-t}\,dt + \int_{14}^{72} 3200*(1+i)^{-t}\,dt;$$

```
In[2]:= i = 0.01;

In[3]:= Solve[NPV]
Out[3]= Solve[2473.39]

In[4]:= i = 0.015;

In[5]:= Solve[NPV]
Out[5]= Solve[-3939.53]

In[6]:= FindRoot[NPV == 0, {i, 0.01, 0.015}]
Out[6]= {0.015 → 0.0117591}
```